II. *Of the* Figure of the Earth, *and the* Variation of Gravity on the Surface. *By Mr.* James Stirling, *F. R. S.* [*See* TAB. *Fig.* 6.]

THE Centrifugal Force, arifing from the Diurnal Rotation of the Earth, depreffeth it at the Poles, and renders it protuberant at the Equator ; as has been lately advanced by Sir *Ifaac Newton,* and long ago by *Polybius,* according to *Strabo* in the Second Book of his *Geography.* But although it be of an oblate fpheriodical Shape, yet the kind of that Spheroid is not yet difcovered ; and therefore I fhall fuppofe it to be the common Spheroid gene-rated by the Rotation of an Ellipfis about its leffer Axis ; although I find by Computation, that it is only nearly, and not accurately fuch. I fhall alfo fup-pofe the Denfity to be every where the fame, from the Center to the Surface, and the mutual Gravita-tion of the Particles towards one another to decreafe in the duplicate Ratio of their Diftances : And then the following Rules will follow from the nature of the Spheroid.

1. Let A D B E be the Meridian of an oblate Spheroid, D E the Axis, A B the Diameter of the Equator, and C the Center. Take any Point on the Surface, as F, from which draw F C to the Center, F G, perpendicular to the Surface at F, meeting C B in G, and F H cutting the Line C G, fo that C H may be to G H as three to two. I fay that a Body at F will gravitate in the Direction F H ; and

and that the mean Force of Gravity on the Surface will be to the Excefs of the Gravity at the Pole above that at F, as the mean Diameter multiplied into the Square of the Radius is to one fifth of the Difference of the longeft and fhorteft Diameters multiplied into the Square of the Co-fine of Latitude at F.

2. The Decrement of Gravity from the Pole to the Equator is proportional to the Square of the Co-fine of Latitude; or, which comes to the fame, the Increment of Gravity from the Equator to the Pole is proportional to the Sine of Latitude. Hitherto I have confider'd the Variation of Gravity which arifes from the fpheroidical Figure, while it does not turn round its Axis ; but if it doth, the Direction of Gravity will be in the Line F G, perpendicular to the Surface ; and its Variation now arifing from both the Figure and centrifugal Force, will be five times greater than what arifes from the Figure alone ; as will appear from the Proportion of the Lines F H and F G, the former being to the latter, as the whole Force of Gravity at F, while the Speroid is at reft, to the Force with which a Body defcends at F, while it turns round its Axis.

3. From this laft Article it appears, that one fifth of the Variation of Gravity is occafion'd by the Figure of the Spheroid, and the remaining four fifths by the centrifugal Force. And whereas the Earth could not be of an oblate fpheroidical Figure, unlefs it turned round its Axis ; nor could it turn round its Axis, without putting on that Figure : I fay, that the Diminution of Gravity towards the Equator, known by the Experiments with Pendu-
lums,

lums, prove both the Rotation and oblate fpheroi-
dical Figure of the Earth.

4. The mean Force of Gravity on the Surface
is to the centrifugal Force at any Point F, as a
Rectangle under the Radius and mean Diameter to
a Rectangle under the Co-fine of Latitude, and four
fifths of the Difference of the longeft and fhorteft
Diameters. And at the Equator, where the Co-fine
of Latitude becomes equal to the Radius, the mean
Force of Gravity is to the centrifugal Force, as the
mean Diameter to four fifths of the longeft and
fhorteft Diameters. This Article is found from the
Proportion of the Lines F H and G H ; the former
being to the latter as the Force of Gravity to the
centrifugal Force.

5. The Proportion of the Diameters of the
Earth will be found in the following manner : The
Moon revolves about the Earth in 27^d, 7^h, 43$^\prime$, or
in 39343 Minutes : And her mean Diftance is a-
bout 59½ Semidiameters of the Earth, according to
La Hire's and *Flamftead*'s Tables ; but near 60½
by *Halley*'s Tables. I fhall therefore take 60 for
the mean Diftance, till it be better known : Then
according to the Nature of Gravity, as the Cube
of the Moons Diftance to the Semidiameter of the
Earth, or as 216000 to Unity, fo is 1547870000 the
Square of the periodick Time of the Moon to 7166,
the Square of the Number of Minutes in which ano-
ther Moon would revolve about the Earth at the
Diftance of its Semidiameter. And as this laft
Number to 2062096, the Square of 1436, the Num-
ber of Minutes in a Sydereal Day, fo is Unity to
287.7 ; which would fhew the Proportion of the
centri-

centrifugal Force at the Equator to the mean Force of Gravity (by *Corol.* 2. *Prop.* 4. *Lib.* 1. *Princip.*) were it not for the Action of the Sun on the Moon. Therefore (by *Corol.* 17. *Prop.* 66. *Lib.* 1. *Princip.*) I fay, as the Square of the Sydereal Year, to the Square of the periodick Time of the Moon, that is, as 179 to Unity, fo is 287.7 to 1.6 ; which being added to 287.7, makes 289.3. And therefore, as Unity to 289, neglecting the Fraction which is uncertain ; fo is the centrifugal Force at the Equator to the mean Force of Gravity on the Surface. And thence (by Article 4.) as 289 to $\frac{1}{4}$, fo is the mean Diameter to the Difference of the longeft and fhorteft : And therefore, as the Axis is to the equatoreal Diameter, fo is 2307 to 2317, or in fmaller Numbers, as 231 to 232, the fame as Sir *Ifaac Newton* found in a different manner ; for he makes it as 230 to 231, and as 230 to 231, fo is 231 to 232. 004.

6. In the fame manner the Proportion of the Diameters of any Planet may be found, if it has a Satellite : For Inftance, in *Jupiter*, he turns about his Axis in 9^h, 56', or in 596 Minutes : And his third Satellite revolves about him in 7^d, 3^h, 42', 36'', or in 10302.6 Minutes, at the diftance of 15.141 of his Semidiameters. Therefore, I fay, as the Cube of 15.141 to Unity, fo is the Square of 10302.6 to 30579, the Square of the Number of Minutes in which a Satellite would revolve about him at the diftance of his Semidiameter : And as this laft Number is to 355216, the Square of 596, fo is Unity to 11$\frac{5}{8}$, or the centrifugal Force at his Equator to the mean Force of Gravity on his Surface. There is no need of correcting this Number,

as

as in the former Article, becaufe the periodick Time
of *Jupiter* round the Sun is vaftly greater than
that of his third Satellite round him. I have chofe
the third Satellite before any of the reft, becaufe its
greateft Elongation was obferv'd by Dr. *Pound*,
with a Micrometer adapted to a Telefcope 123 Feet
long ; and he alfo took the Diameter of *Jupiter*
by the Tranfit of the Satellite, which is a much
more exact Way than with a Micrometer. But as
the Planes of *Jupiter*'s Satellities almoft coincide
with the Plane of his Equator, the Diameter, de-
termined by the Tranfit of the Satellite, is his
greateft ; and the Diftance of the Satellite, which
ought to have been given in his mean Diameters, is
affigned in his greateft : For which Reafon the Force
of Gravity already found, muft be augmented in the
triplicate Ratio of his greateft Diameter to his mean
one ; that is, if a reprefent the mean Diameter, and
d the Difference of the longeft and fhorteft, in the
Proportion of $2a + 3d$ to $2a$ very nearly. Hence,
as the centrifugal Force at his Equator, to the mean
Force of Gravity on his Surface, fo is Unity to
$11\frac{5}{8} \times \frac{2a + 3d}{2a}$. And (by Article 4.) $11\frac{5}{8} \times \frac{2a + 3d}{2a} : 1$
$:: a : \frac{4}{5}d$, or $20aa = 186ad + 279dd$; which
makes a to d, as 108 to 10 ; and thence the Axis is
to the equatoreal Diameter, as $108 - 5$ to $108 + 5$,
or as 103 to 113 ; that is, as 12 to $13\frac{1}{6}$: Which
agrees nicely with the Obfervations of both Dr.
Pound and Mr. *Bradley*, made with *Huygens*'s Long
Telefcope ; the former making it as 12 to 13, and
the latter as 25 to 27, which is very nearly the
fame. And if this Theory agrees fo well with Ob-
fervations

fervations in *Jupiter*, there is no doubt but it will
be more exact in the Earth, whofe Diameters are
much nearer to Equality.

7. By Experiments made at *Jamaica* * in the
Latitude of 18 Degrees with a very curious Clock,
contriv'd by Mr. *Graham*, it was found that the
London Pendulum went flower there by 2' 6" in a
Sydereal Day, than at *London*. But it was found
by Experiments made with Thermometers, that 9"
were to be allowed for the lengthening of the Pen-
dulum by Heat ; and therefore it was retarded
only 1' 57" by the Decrement of Gravity. So
that while a Pendulum of *London* makes 86164 Vi-
brations, the Number of Seconds in a Sydereal
Day, the fame at *Jamaica* only gives 86047 Vibra-
tions. Therefore the Force of Gravity at *London*
is to that in the Latitude of 18 Degrees, as the
Square of 86164 to the Square of 86047 ; that is,
very nearly as 1106 to 1103. And (by Article 1,
and 2.) if *a* denote the mean Diameter of the
Earth, *d* the Difference of the greateft and fmalleft ;

$a - \dfrac{ccd}{rr}$ will denote the Force of Gravity in general

in any Latitude, whofe Co-fine is to the Radius
as *c* to *r :* Where, if in the Place of *c* there be
fubftituted the Co-fines of 51°: 32' and 18°: 0',
that is of the Latitudes of *London* and *Jamaica*,
we fhall have the Force of Gravity at the former
to that at the latter, as $a - \lfloor 3870\, d$ to $a - \lfloor 9045\, d$,
that is as 1106 to 1103. Whence the mean Di-
ameter of the Earth will be to the Difference of the

* See *Tranfact.* N°· 432. p. 302.

Axis

Axis and equatoreal Diameter, as 191 to Unity ; and thence (by Article 4.) as the mean Gravity on the Surface to the centrifugal Force at the Equator, fo is 191 to ⁴⁄₄, or fo is 239 to Unity. In order to fhew that this cannot be, I fhall obferve, that when the Moon's Diftance was fuppofed 60 Semidiameters of the Earth (as in Article 5.) it was found that the mean Force of Gravity was to the centrifugal Force at the Equator, as ₂89 to 1. But if the Proportion now found be true, the Moon's Diftance of 60 Semi-diameters muft be augmented in the fubtriplicate Proportion of 289 to 239, and then it will become 64 Semidiameters. In the like manner, if we com-pute the Ratio of the mean Force of Gravity to the centrifugal Force, by prefuppofing the Mag-nitude of the Earth, as Sir *Ifaac Newton* and Mr. *Hugens* did, we muft fuppofe a Degree to be above 80 *Englifh* Miles to bring it out 239 to Unity. Now whereas it is certain that the Diftance of the Moon is about 60 Semidiameters of the Earth, and that a Degree is lefs than 70 *Englifh* Miles ; therefore, I fay, that the Conclufion which feems to follow from the *Jamaica* Experiment, cannot be allowed to be true. And the Experi-ments made by *Richer*, in the Ifland of *Cayenna*, would ftill make a greater Difference betwixt the Diameters of the Earth, than thofe made in *Jamaica*. And the Lengths of the *Paris* and *London* Pendu-lums compared together, would make it greater than one 231 Part of the Whole, as it was found in Article 5.

8. From all the Experiments made with Pendu-lums, it appears that the Theory makes them longer

in

in Iflands, than they are found in fact. The *London* Pendulum fhould be longer when compared to the *Paris* one, than it really is : The *Jamaica* Pendulum, when compared to the *London* one, which vibrates in a greater Ifland, fhould be longer than is found by Experience ; and the Pendulum in *Cayenna* (a fmaller Ifland than *Jamaica*) fhould ftill be longer. This Defect of Gravity in Iflands is very probably occafioned by the Vicinity of a great Quantity of Water, which being fpecifically lighter than Land, attracts lefs in Proportion to its Bulk. And I find by Computation, that the Odds in the Pendulums betwixt Theory and Practice is not greater than what may be accounted for on that Suppofition. I fhall alfo obferve, that although the Matter of the Earth were entirely uniform, yet the Hypothefis of its being a true Spheroid is not near enough the Truth to give the Number of Vibrations which a Pendulum makes in twenty-four Hours. And fuppofe the true Figure were known, the Inequalities of Mountains and Vallies, Land and Water, Heat and Cold, would never allow Theory and Experiments to agree. But after the *French* Gentlemen who are now about meafuring a Degree, and making Experiments with Pendulums in the North and South, fhall have finifhed their Defign, we may expect new Light in this Matter.

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Fig.1

Fig. 3

Fig. 2

Fig. 1

Fig. 4.

Fig. 6

Fig.5